Scale, Proportion, and Quantity

In thinking scientifically about systems and processes, it is essential to recognize that they vary in size (e.g., cells, whales, galaxies), in time span (e.g., nanoseconds, hours, millennia), in the amount of energy flowing through them (e.g., lightbulbs, power grids, the sun), and in the relationships between the scales of these different quantities. The understanding of relative magnitude is only a starting point. As noted in *Benchmarks for Science Literacy*, "The large idea is that the way in which things work may change with scale. Different aspects of nature change at different rates with changes in scale, and so the relationships among them change, too" [4]. Appropriate understanding of scale relationships is critical as well to engineering—no structure could be conceived, much less constructed, without the engineer's precise sense of scale.

From a human perspective, one can separate three major scales at which to study science: (1) macroscopic scales that are directly observable—that is, what one can see, touch, feel, or manipulate; (2) scales that are too small or fast to observe directly; and (3) those that are too large or too slow. Objects at the atomic scale, for example, may be described with simple models, but the size of atoms and the number of atoms in a system involve magnitudes that are difficult to imagine. At the other extreme, science deals in scales that are equally difficult to imagine because they are so large—continents that move, for example, and galaxies in which the nearest star is 4 years away traveling at the speed of light. As size scales change, so do time scales. Thus, when considering large entities such as mountain ranges, one typically needs to consider change that occurs over long periods. Conversely, changes in a small-scale system, such as a cell, are viewed over much shorter times. However, it is important to recognize that processes that occur locally and on short time scales can have long-term and large-scale impacts as well.

In forming a concept of the very small and the very large, whether in space or time, it is important to have a sense not only of relative scale sizes but also of what concepts are meaningful at what scale. For example, the concept of solid matter is meaningless at the subatomic scale, and the concept that light takes time to travel a given distance becomes more important as one considers large distances across the universe.

Understanding scale requires some insight into measurement and an ability to think in terms of orders of magnitude—for example, to comprehend the difference between one in a hundred and a few parts per billion. At a basic level, in order to identify something as bigger or smaller than something else—and how much bigger or smaller—a student must appreciate the units used to measure it and develop a feel for quantity.

The ideas of ratio and proportionality as used in science can extend and challenge students' mathematical understanding of these concepts. To appreciate the relative magnitude of some properties or processes, it may be necessary to grasp the relationships among different types of quantities—for example, speed as the ratio of distance traveled to time taken, density as a ratio of mass to volume. This use of ratio is quite different than a ratio of numbers describing fractions of a pie. Recognition of such relationships among different quantities is a key step in forming mathematical models that interpret scientific data.

Progression

The concept of scale builds from the early grades as an essential element of understanding phenomena. Young children can begin understanding scale with objects, space, and time related to their world and with explicit scale models and maps. They may discuss relative scales—the biggest and smallest, hottest and coolest, fastest and slowest—without reference to particular units of measurement.

Typically, units of measurement are first introduced in the context of length, in which students can recognize the need for a common unit of measure—even develop their own before being introduced to standard units—through appropriately constructed experiences. Engineering design activities involving scale diagrams and models can support students in developing facility with this important concept.

Once students become familiar with measurements of length, they can expand their understanding of scale and of the need for units that express quantities of weight, time, temperature, and other variables. They can also develop an understanding of estimation across scales and contexts, which is important for making sense of data. As students become more sophisticated, the use of estimation can help them not only to develop a sense of the size and time scales relevant to various objects, systems, and processes but also to consider whether a numerical result sounds reasonable. Students acquire the ability as well to move back and forth between models at various scales, depending on the question being considered. They should develop a sense of the powers-of-10 scales and what phenomena correspond to what scale, from the size of the nucleus of an atom to the size of the galaxy and beyond.

Well-designed instruction is needed if students are to assign meaning to the types of ratios and proportional relationships they encounter in science. Thus the ability to recognize mathematical relationships between quantities should begin developing in the early grades with students' representations of counting (e.g., leaves on a branch), comparisons of amounts (e.g., of flowers on different plants), measurements (e.g., the height of a plant), and the ordering of quantities such as number, length, and weight. Students can then explore more sophisticated mathematical representations, such as the use of graphs to represent data collected. The interpretation of these graphs may be, for example, that a plant gets bigger as time passes or that the hours of daylight decrease and increase across the months.

As students deepen their understanding of algebraic thinking, they should be able to apply it to examine their scientific data to predict the effect of a change in one variable on another, for example, or to appreciate the difference between linear growth and exponential growth. As their thinking advances, so too should their ability to recognize and apply more complex mathematical and statistical relationships in science. A sense of numerical quantity is an important part of the general "numeracy" (mathematics literacy) that is needed to interpret such relationships.